

Gluon fragmentation into charmonium at NLO

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Quarkonium Production in Elementary and Heavy Ion Collisions

17 June 2011

Outline

Part I. **Fragmentation** into heavy quarkonium
from early predictions to latest developments

Part II. **Gluon fragmentation** into charmonium at **NLO**
work in progress

Reference

Eric Braaten

Quarkonium Production via Fragmentation Revisited

talk given at the workshop

“Quarkonium production, Probing QCD at the LHC”

17-21 April 2011, Vienna University of Technology

I. **Fragmentation** into heavy quarkonium
from early predictions to latest developments

Quarkonium production

I. Creation of heavy quark and antiquark

- what are the relevant parton processes?
- can they be calculated
using perturbative QCD
in terms of α_s and m_Q ?

2. Binding of $Q\bar{Q}$ to form quarkonium

- can it be parametrized by a few functions
or (better yet) by a few constants?

Quarkonium production

I. Creation of heavy quark and antiquark

- what are the relevant **parton processes**?
- can they be calculated using **perturbative QCD** in terms of α_s and m_Q ?

related Q:
fragmentation
or
complete fixed-order ?

2. Binding of $Q\bar{Q}$ to form quarkonium

- can it be parametrized by a few **functions** or (better yet) by a few **constants**?

PQCD Factorization Theorem

Collins & Soper 1982

production of a single **hadron**
with **large transverse momentum**
is dominated by fragmentation

- **hard scattering** produces **parton** with larger momentum
- **parton** hadronizes into a **jet** that includes the **hadron**
- **factorization formula**: proved rigorously to all orders in α_s

PQCD Factorization Theorem

Collins & Soper 1982

$$d\sigma[H(P)] = \sum_i \int_0^1 dz \, d\hat{\sigma}[i(P/z)] D_{i \rightarrow H}(z) + \mathcal{O}(\Lambda_{\text{QCD}}^2/p_T^2)$$

- sum over **partons** i
integral over momentum fraction z
- **cross section** $d\hat{\sigma}$ for **parton** with larger momentum P/z
calculate using **PQCD** as power series in $\alpha_s(p_T/z)$
- **fragmentation** function $D_{i \rightarrow H}(z)$
probability for **hadron** to carry **fraction** z of **parton** momentum
nonperturbative function, but logarithmic evolution with p_T is **perturbative**

Application to charmonium

Inclusive cross section for charmonium with $p_T \gg m_c$ factors!

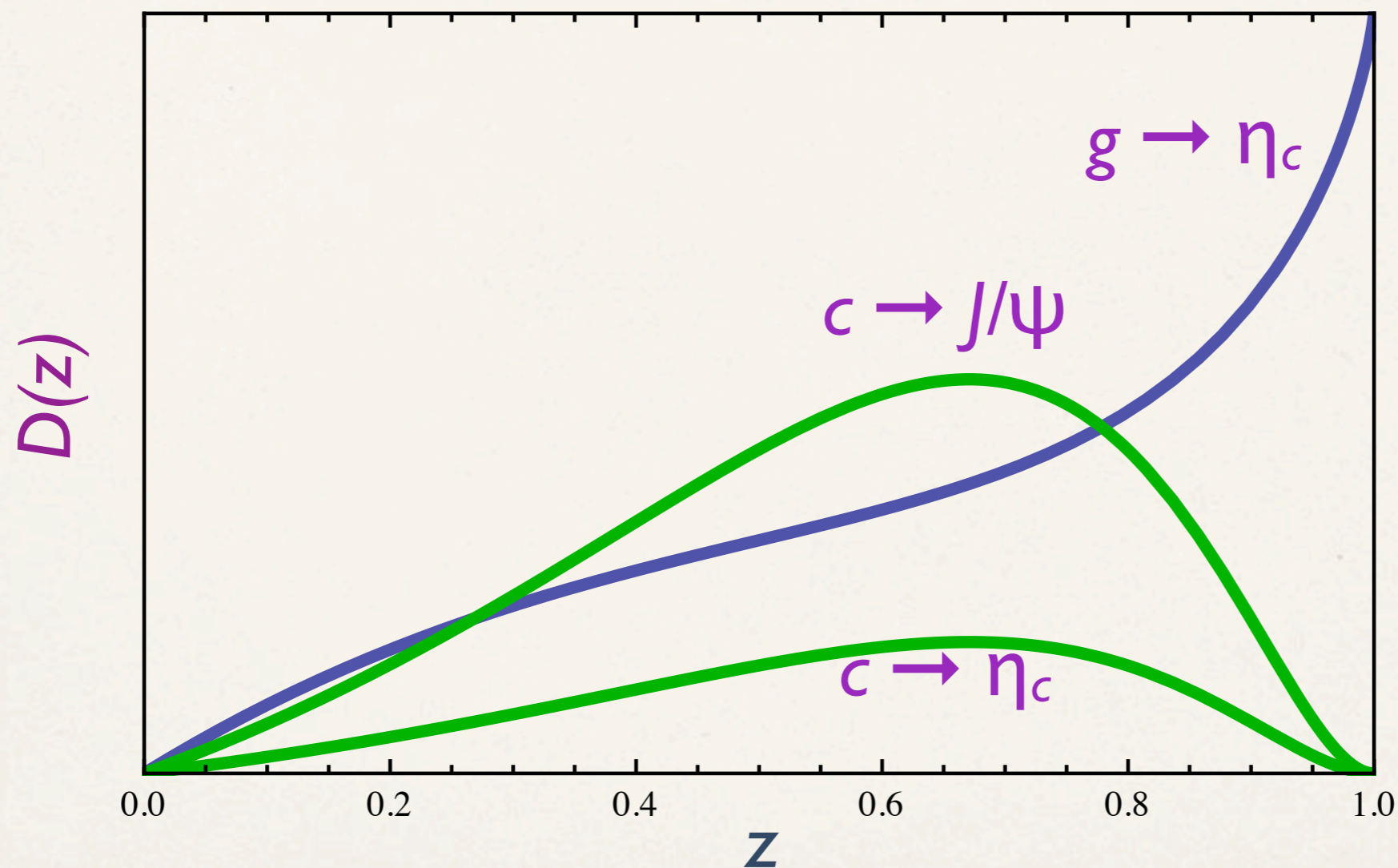
$$d\sigma[H(P)] = \sum_i \int_0^1 dz \, d\hat{\sigma}[i(P/z)] D_{i \rightarrow H}(z) + \mathcal{O}(m_c^2/p_T^2)$$

- cross section $d\hat{\sigma}$ for parton ($i = c, \bar{c}, g, \dots$)
with larger momentum P/z
calculate using PQCD as power series in $\alpha_s(p_T/z)$
- fragmentation function $D_{i \rightarrow H}(z)$
probability for charmonium to carry fraction z
of momentum of jet initiated by parton i
nonperturbative (but not completely)
logarithmic evolution with p_T is perturbative
involves hard momentum scale m_c and softer scales

Parton fragmentation

- fragmentation functions $D_{i \rightarrow H}(z)$ for S-wave charmonium can be calculated using PQCD in Color-Singlet Model

Braaten, Cheung, and Yuan 1993



- reduces nonperturbative functions $D_{i \rightarrow H}(z)$ to nonperturbative constants f_H

Parton fragmentation vs LO (in CSM)

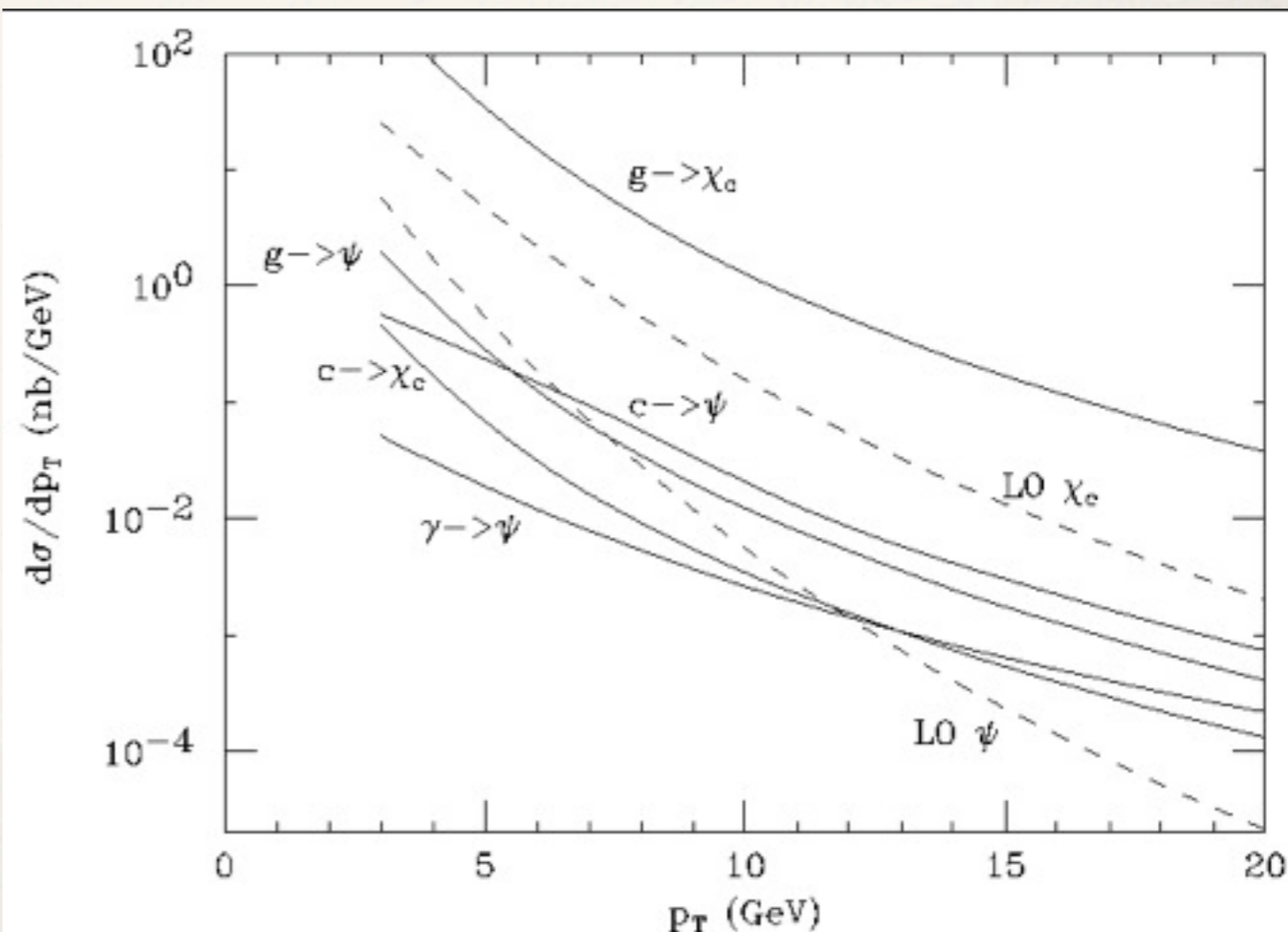
- fragmentation changes the dependence on p_T at large p_T

behavior of $p_T^4 d\hat{\sigma}/dp_T^2$ from gluon-gluon collisions
in the Color-Singlet Model

	LO in α_s	fragmentation
$\eta_c, \chi_{c0}, \chi_{c2}$	$\alpha_s^3 m_c^2/p_T^2$	$\alpha_s(p_T)^2 \alpha_s(m_c)^2$
$J/\psi, h_c, \chi_{c1}$	$\alpha_s^3 m_c^4/p_T^4$	$\alpha_s(p_T)^2 \alpha_s(m_c)^3$

- fragmentation dominates
over LO in α_s
for charmonium at large p_T
at the Tevatron

Doncheski, Fleming
& Mangano 1994

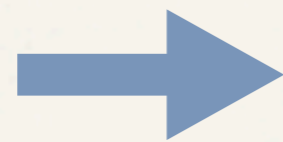


Parton fragmentation in the CSM

Two problems:

1. infrared divergences for P-waves

fragmentation functions for $g \rightarrow \chi_{cJ}$
are infrared divergent at LO in α_s



CSM is inconsistent for P-wave,
IR divergence cancelled in NRQCD

2. delayed accuracy of the fragmentation approximation:

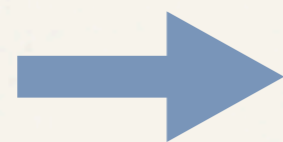
for specific production channels, a reasonable accuracy
is reached only at very large p_T , i.e. in a region that is
not accessible experimentally

Parton fragmentation in the CSM

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1. infrared divergences for P-waves

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CSM is inconsistent for P-wave,
IR divergence cancelled in NRQCD

see next
slides

2. delayed accuracy of the fragmentation approximation:

for specific production channels, a reasonable accuracy
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not accessible experimentally

I come back to this
point later on

NRQCD factorization

Bodwin, Braaten & Lepage 1995

Conjectured factorization formula

for inclusive production of charmonium H

motivated by perturbative QCD factorization theorems
and by effective field theory

$$d\sigma[H] = \sum_n d\hat{\sigma}[c\bar{c}(n)] \langle \mathcal{O}_n^H \rangle$$

- sum over color/angular momentum channels
1 or 8 $^1S_0, ^3S_1, ^1P_1, ^3P_0, ^3P_1, ^3P_2, \dots$
- parton cross sections for creating $c\bar{c}$
expand in powers of $\alpha_s(m_c)$
- NRQCD matrix elements for formation of H
scale as definite powers of v

NRQCD factorization

- For S-wave charmonium states:
truncation at leading-order in v reproduces
the Color-Singlet Model
- For P-wave charmonium states:
infrared divergence problem is solved by
adding color-octet terms of leading order in v

NRQCD factorization

For pheno purposes, the following truncation of the expansion in v may be accurate:

- for **S-waves**, truncate after order v^7

$$J/\psi : \quad \langle \underline{1} \ ^3S_1 \rangle \sim v^3$$

$$\langle \underline{8} \ ^3P_J \rangle, \langle \underline{8} \ ^1S_0 \rangle, \langle \underline{8} \ ^3S_1 \rangle \sim v^7$$

\Rightarrow 4 universal constants for $J/\psi, \eta_c$

(I determined by $J/\psi \rightarrow l^+l^-$)

- for **P-waves**, truncate after order v^5

$$\chi_{cJ} : \quad \langle \underline{1} \ ^3P_J \rangle, \langle \underline{8} \ ^3S_1 \rangle \sim v^5$$

\Rightarrow 2 universal constants for $\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c$

(I determined by $\chi_{c0} \rightarrow \gamma\gamma$)

Inclusive quarkonium production

Theoretical status ~ 2000

- rigorous **factorization theorem** for $p_T \gg m_Q$
hadronization described by
nonperturbative fragmentation functions $D_{i \rightarrow H}(z)$

$$d\sigma[H(P)] = \sum_i \int_0^1 dz \, d\hat{\sigma}[i(P/z)] D_{i \rightarrow H}(z) + \mathcal{O}(m_c^2/p_T^2)$$

- **NRQCD factorization formula**
hadronization described by
hierarchy of **nonperturbative constants**

$$D_{i \rightarrow H}(z) = \sum_n d_{g \rightarrow c\bar{c}[n]}(z) \langle \mathcal{O}_n^H \rangle$$

Inclusive quarkonium production

Theoretical status ~ 2000

- rigorous factorization theorem for $p_T \gg m_Q$
hadronization described by

nonperturbative fragmentation function
Exp: production rate is measured in
in a limited p_T region

$$d\sigma[H(P)] = \sum_i \int_0^1 d\alpha$$

Th: accuracy of the fragmentation approximation in this region ?

$$+ \mathcal{O}(m_c^2/p_T^2)$$

- NRQCD factorization formula
hadronization described by
hierarchy of nonperturbative constants

$$D_{i \rightarrow H}(z) = \sum_n d_{g \rightarrow c\bar{c}[n]}(z) \langle \mathcal{O}_n^H \rangle$$

Delayed accuracy of the fragmentation approximation

- First mentioned in the case of hadronic production of B_c, B_c^*

$$d\hat{\sigma}[gg \rightarrow B_c + b + \bar{c}] \longrightarrow d\hat{\sigma}[gg \rightarrow \bar{b} + b] \otimes D_{\bar{b} \rightarrow B_c}$$

fragmentation functions for $b \rightarrow B_c, B_c^*$ at LO in α_s

Braaten, Cheung & Yuan 1993

complete calculation of $g g \rightarrow B_c + b + \bar{c}$ at LO in α_s

Chang, Chen, Han & Jiang; Berezhnoy, Likhoded & Shevlyagin; Kolodziej, Leike & Ruckl 1995

for $s_{gg}^{1/2} = 200 \text{ GeV}$, fragmentation approximation

is not accurate until $p_T > 60 \text{ GeV} !$

Chang, Chen & Oakes 1995

- Same situation for the hadronic production of $J/\psi + c\bar{c}$

P.A., J. Lansberg & F. Maltoni 2006

QCD correction to quarkonium production

- In the past few years, **NLO corrections** in α_s to quarkonium production have been computed in the **complete fixed-order scheme** (by opposition with the fragmentation approximation)

$$d\sigma[H] = \sum_n d\hat{\sigma}[c\bar{c}(n)] \langle \mathcal{O}_n^H \rangle$$

In the case of hadroproduction, SD coefficients are known at **NLO accuracy** for **all the channels** involved in the **standard truncation** in v

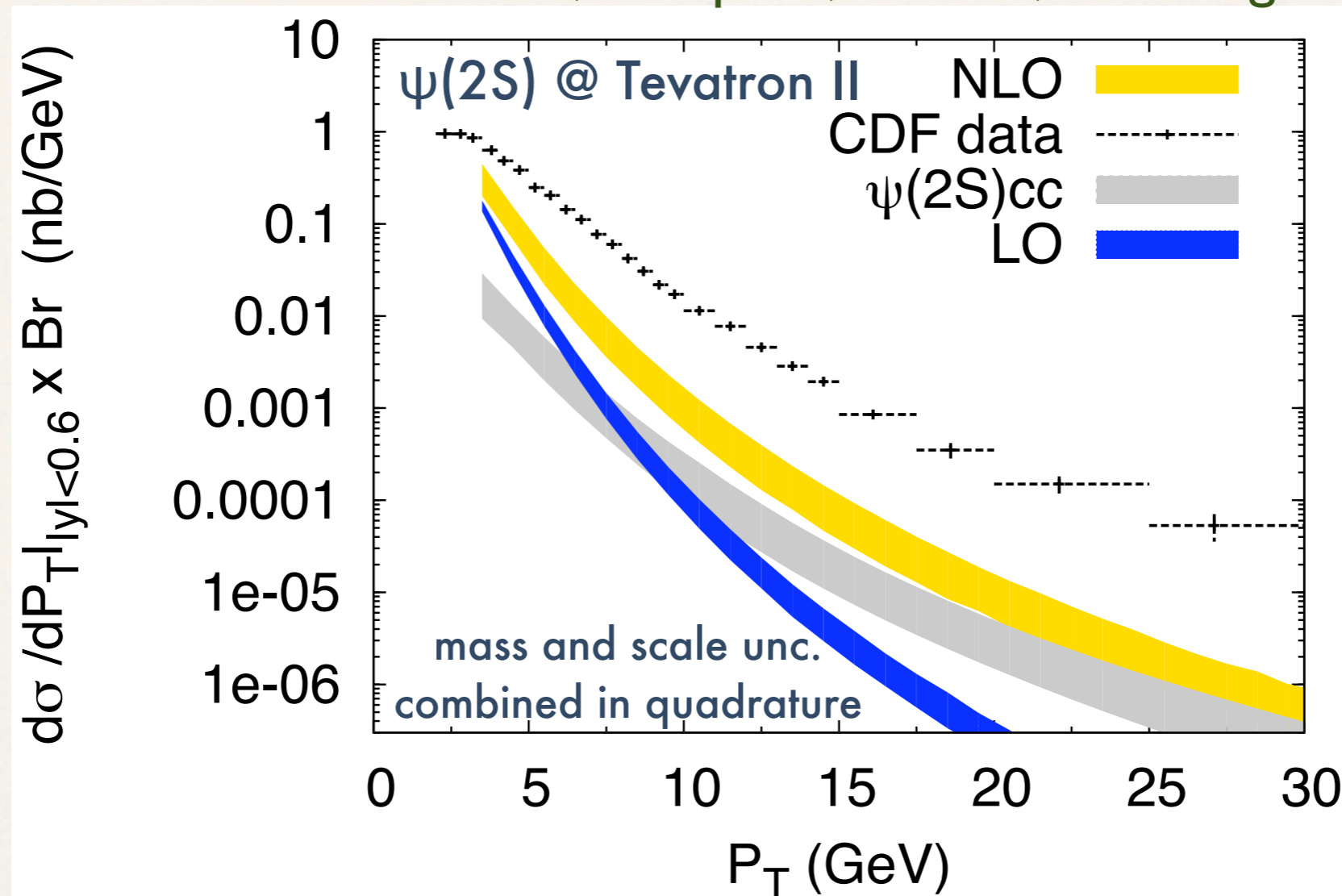
see Geoff Bodwin's talk

NLO correction to color-singlet 3S_1

Campbell, Maltoni, Tramontano, 2007

PA, Campbell, Maltoni, Lansberg & Tramontano, 2007

Gong, Wang; 2007

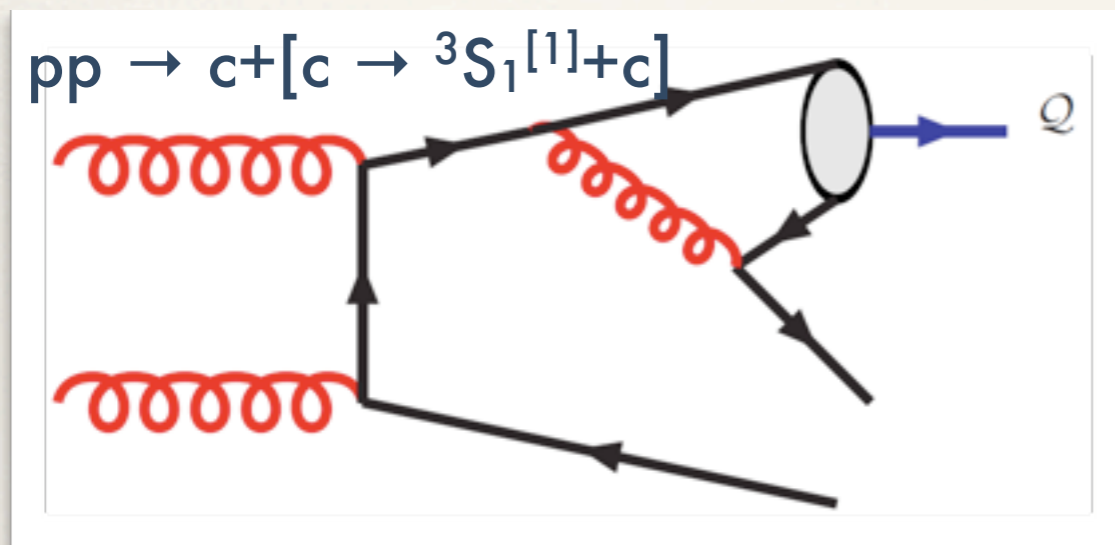


- new channels at α_s^4 give rise to a **huge enhancement at large p_T** , overall the correction is small
- large sensitivity to the renormalization scale

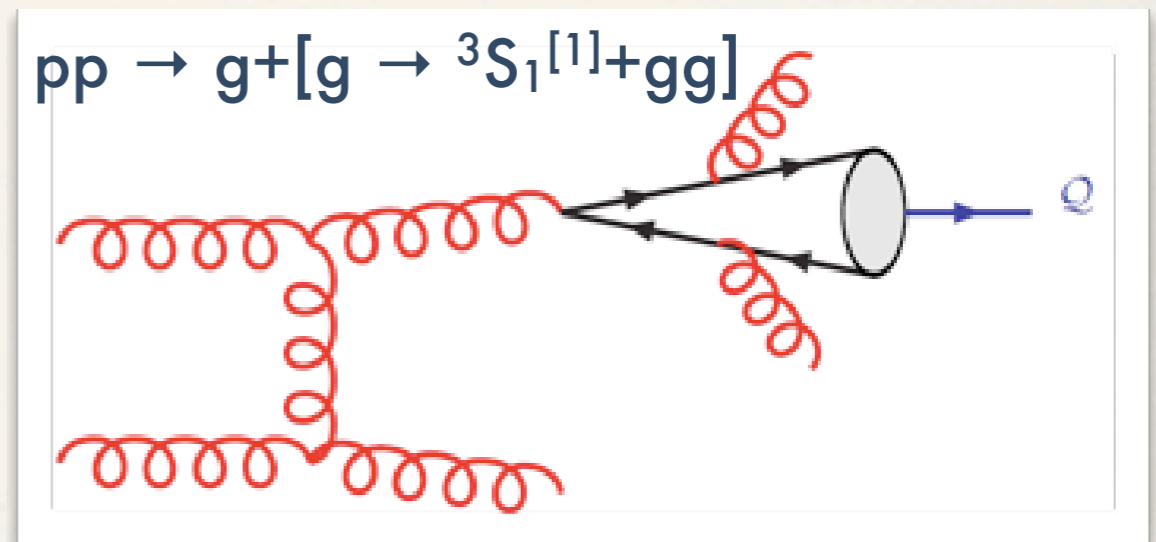
NLO correction to color-singlet 3S_1

In this case, parton fragmentation contributions appears

- at NLO in α_s



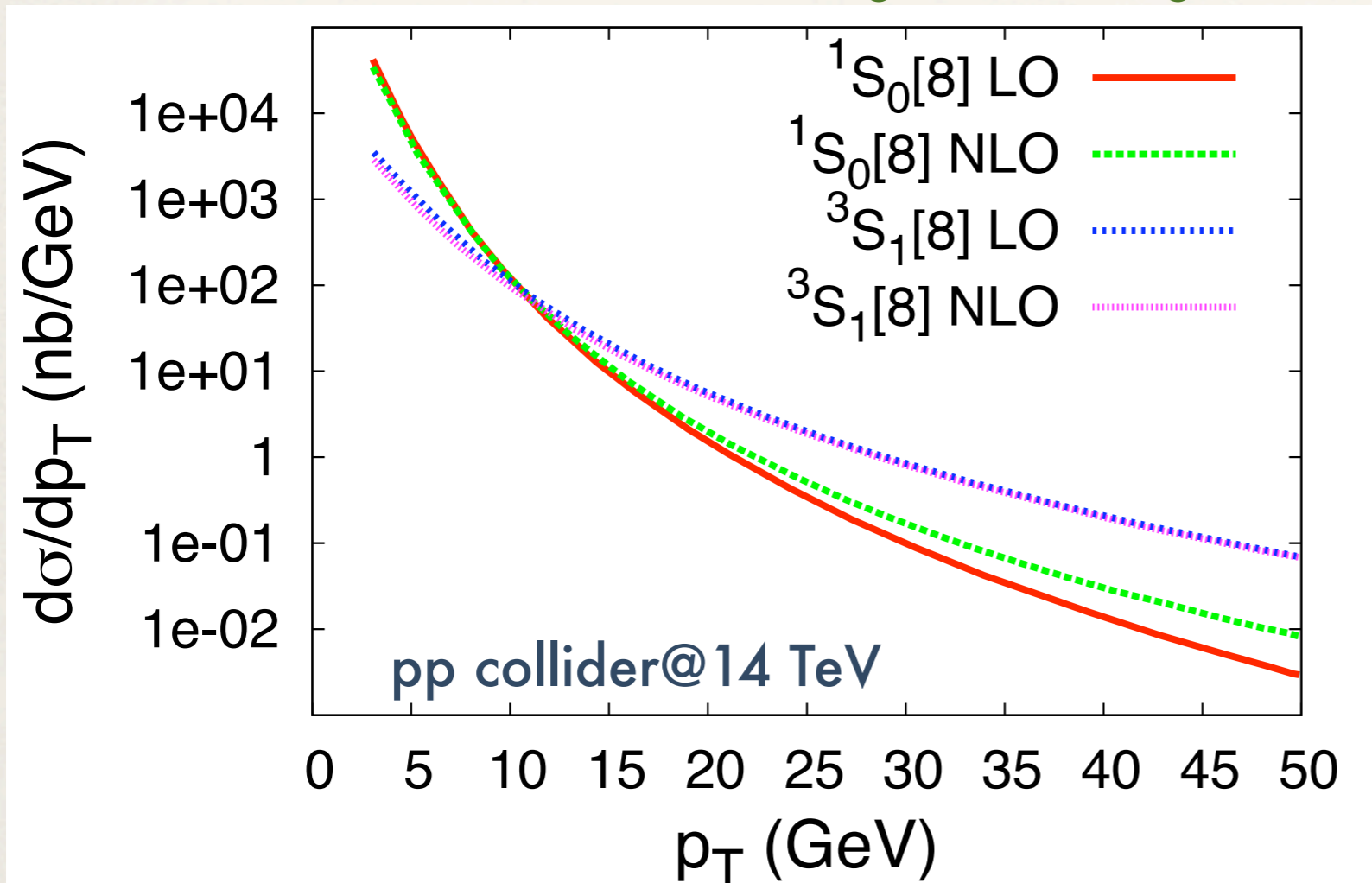
- at NNLO in α_s



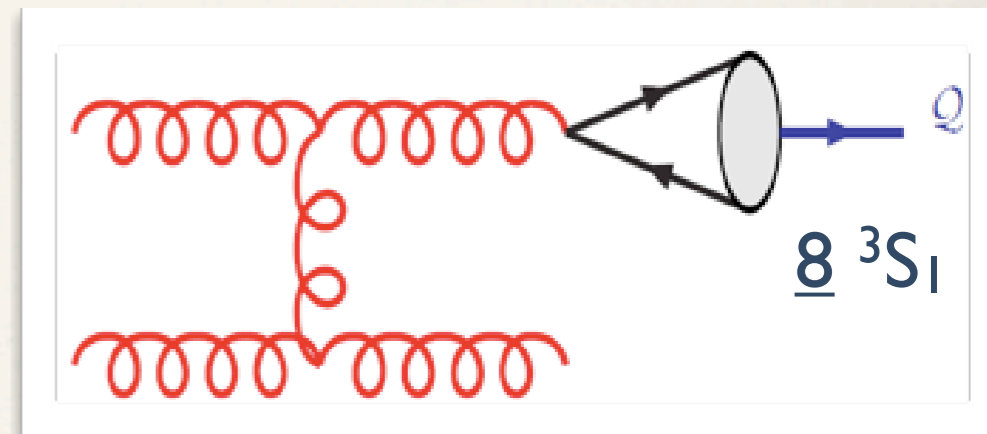
\Rightarrow NLO in α_s is **not NLO** accuracy at large p_T !

NLO correction to color-octet S-wave

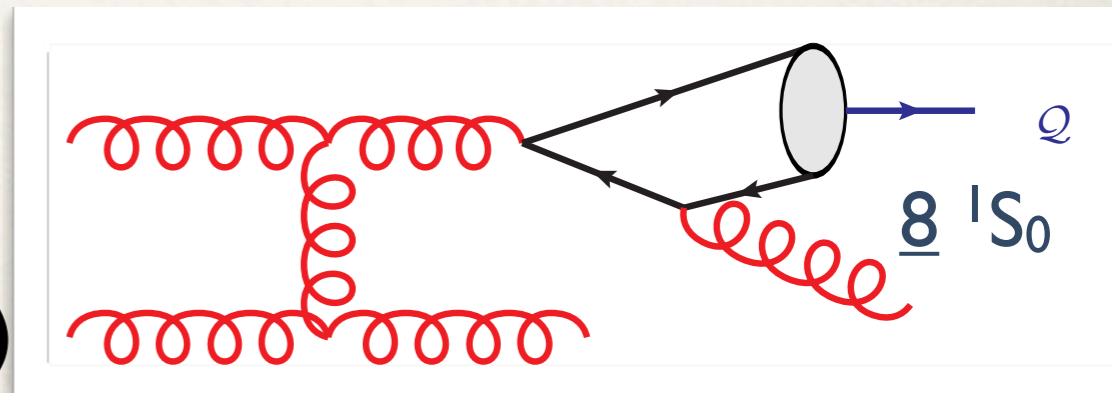
Gong, Li & Wang 2008



8 3S_1 : small K-factor
gluon frag. appears at **LO**



8 1S_0 : large K factor at large p_T
gluon frag. appears at **NLO** in α_s
(as it does for color-octet P-wave)



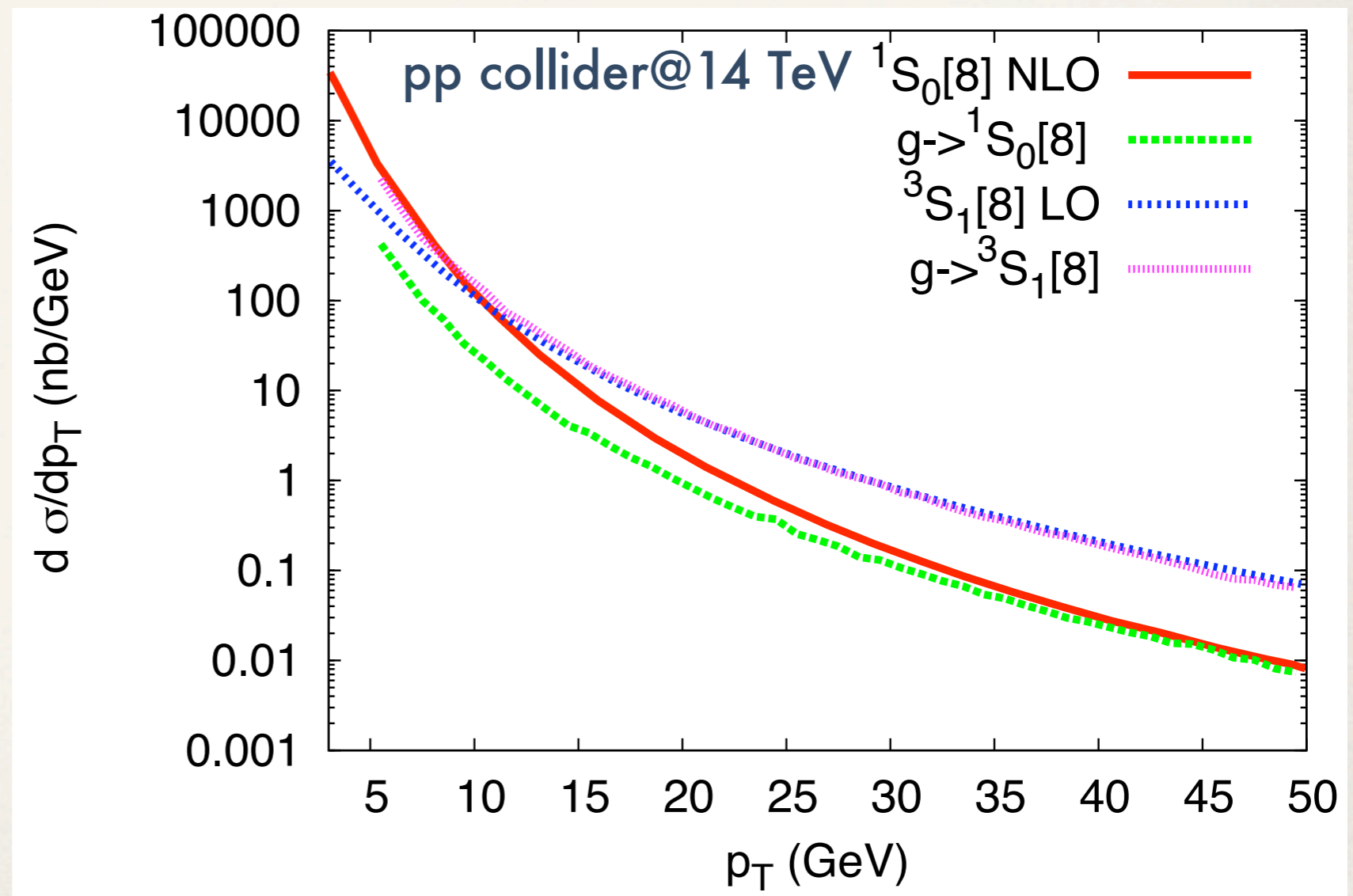
Fragmentation vs complete Fixed-Order

* $\underline{8}^3S_1$: FO LO vs frag. LO $\underline{8}^1S_0$: FO NLO vs frag. LO

(same input parameters, no evolution)

$\underline{8}^3S_1$: small correction
to the frag. approx. in
the range $p_T \gtrsim 7$ GeV

$\underline{8}^1S_0$: small correction
to the frag. approx. in
the range $p_T \gtrsim 30$ GeV



Fragmentation vs complete Fixed-Order

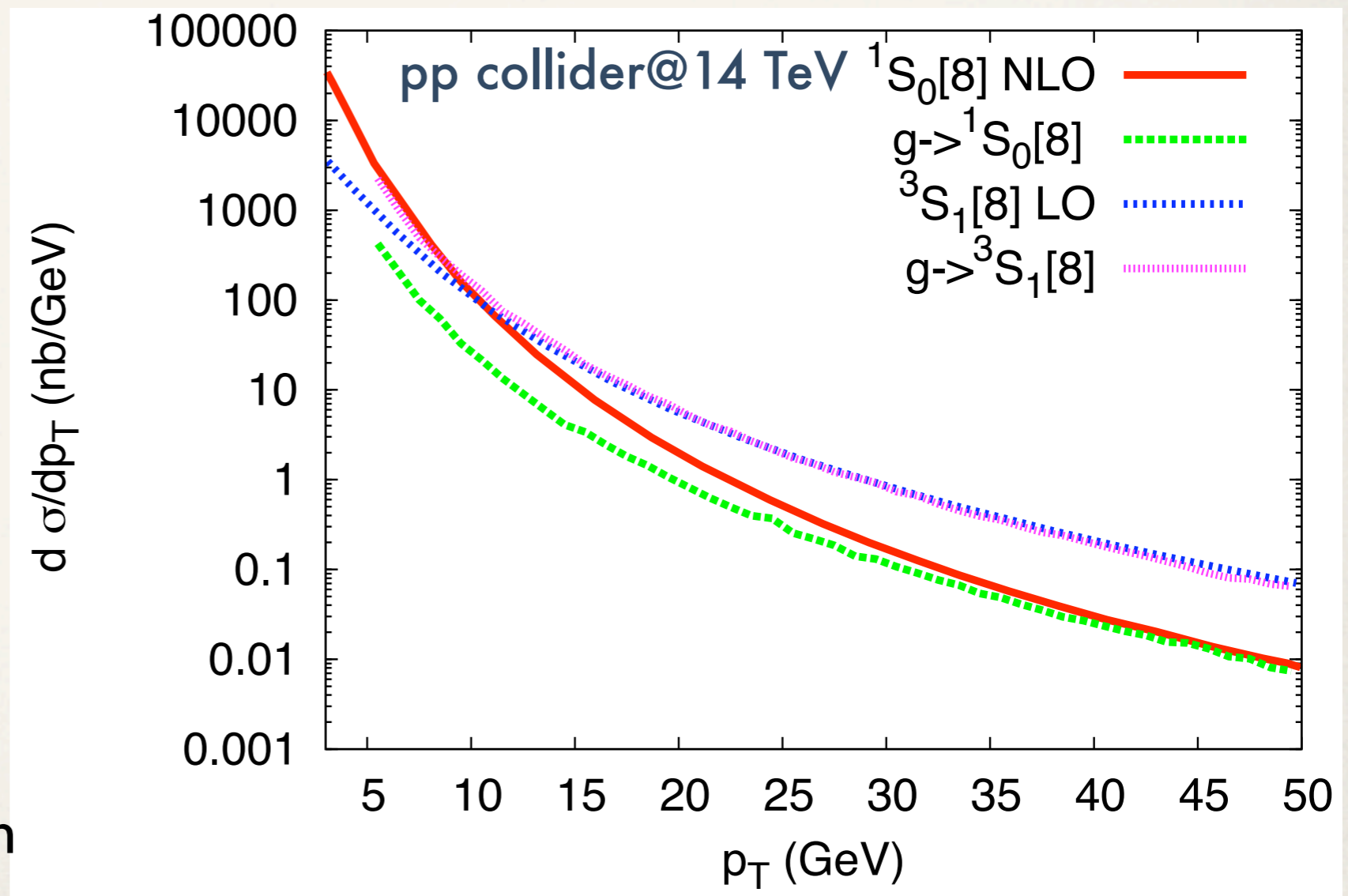
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delayed accuracy of the
fragmentation approximation



So one has the following situation:

Theory

For **very large p_T** , fragmentation is an appealing framework:

1 **factorization** is proven up to NNLO in α_s

2 most **accurate** predictions (potentially): genuine NLO accuracy + log resummation



TENSION

Experiment

Most events are produced in the **low p_T region**

\Rightarrow stat. errors increase with p_T

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Exp. developments:

ATLAS/CMS experiments at the LHC

\Rightarrow access to a larger p_T range

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Most events are produced in the **low p_T region**

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Th. developments:

Fragmentation framework has been extended to take into account $\mathcal{O}(m_Q^2/p_T^2)$ terms

Kang, Qiu, Sterman

$Q\bar{Q}$ Fragmentation

see George Sterman's talk

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Exp. developments:

ATLAS/CMS experiments at the LHC
 \Rightarrow access to a larger p_T range

Does it help to reduce the tension?

e.g.: extract more accurately the LDME

$Q\bar{Q}$ Fragmentation

Rigorous PQCD factorization theorem
for quarkonium production at large p_T
through next-to-leading order in m_Q^2/p_T^2

Kang, Qiu, Sterman

- at leading order in m_Q/p_T ,
fragmentation of single partons (Q, \bar{Q}, g, \dots)

Collins & Soper 1983

- at order m_Q^2/p_T^2 , new mechanism!
 $Q\bar{Q}$ fragmentation into quarkonium

see George Sterman's talk

Q \bar{Q} Fragmentation

New factorization formula

$$\begin{aligned}
 d\sigma[H] = & \sum_i d\hat{\sigma}[i] \otimes D[i \rightarrow H] && \text{LO in } m_c/p_T \\
 & + \sum_m d\hat{\sigma}[Q\bar{Q}_m] \otimes D[Q\bar{Q}_m \rightarrow H] && \text{order } m_c^2/p_T^2 \\
 & + d\sigma_{\text{direct}}[H] && \text{order } m_c^4/p_T^4
 \end{aligned}$$

- cross sections in fragmentation terms: $d\sigma[i]$, $d\sigma[Q\bar{Q}_m]$
 convolutions of
 parton distributions for colliding hadrons
 parton cross sections: calculate as expansions in $\alpha_s(p_T/z)$
- direct cross section $d\sigma_{\text{direct}}[H]$
 remainder after subtracting fragmentation terms
 may not be calculable beyond NLO in $\alpha_s(m_c)$

$Q\bar{Q}$ Fragmentation

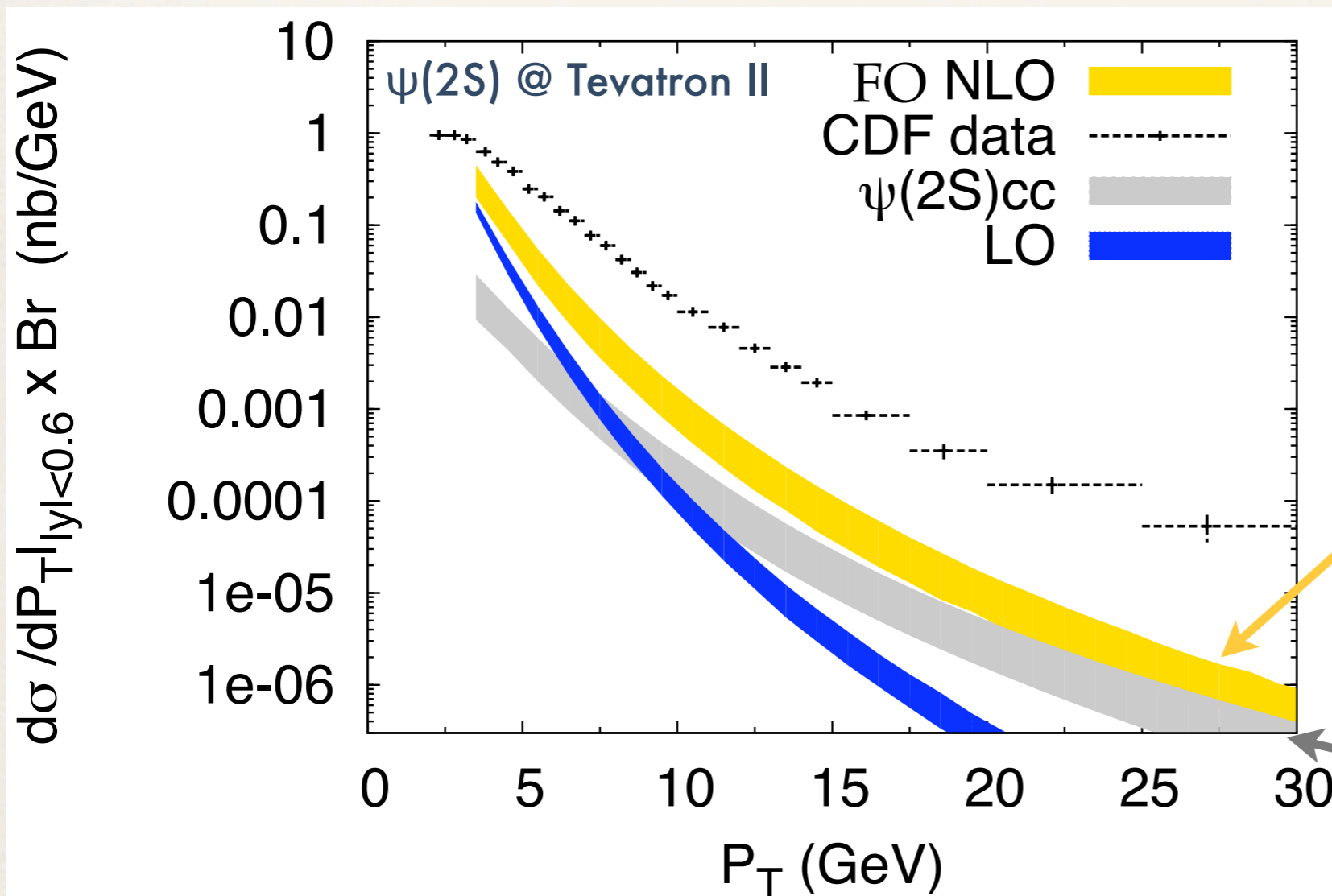
New factorization formula

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 & + d\sigma_{\text{direct}}[H] && \text{order } m_c^4/p_T^4
 \end{aligned}$$

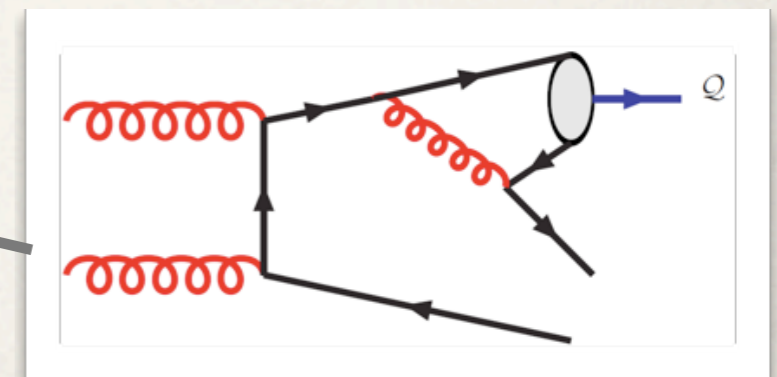
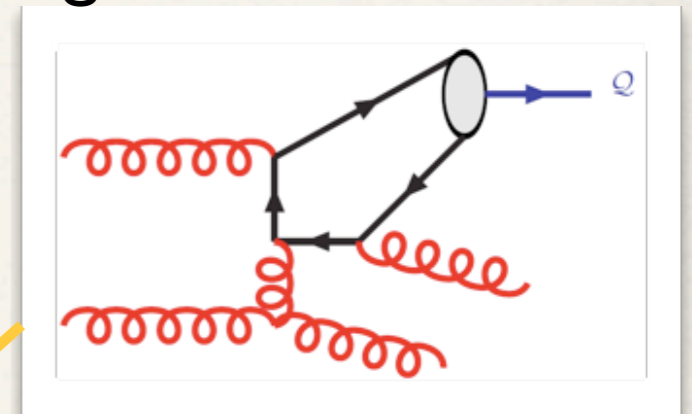
- fragmentation functions: $D_{i \rightarrow H}(z)$, $D_{Q\bar{Q} \rightarrow H}(z, \zeta, \zeta')$
nonperturbative (but not completely)
 logarithmic evolution with p_T is **perturbative**
 involve **hard** momentum scale m_Q as well as **soft** scales
- **NRQCD factorization** can probably be used
 to factor out the remaining **hard** momentum scale m_Q
 and reduce the **nonperturbative functions** to **constants**

Possible impact on the pheno

- At intermediate p_T , $Q\bar{Q}$ fragmentation may be the dominant contribution in the case of $\underline{1}^3S_1$



FO scheme: NLO in α_s
Frag. scheme: LO in α_s



Possible impact on the pheno

- ❖ At intermediate p_T , $Q\bar{Q}$ fragmentation may be the dominant contribution in the case of $\underline{1}^3S_1$
 - \Rightarrow new factorization formalism gives a practical access to the calculation of the p_T spectrum at genuine NLO accuracy over the whole p_T range
- ❖ $Q\bar{Q}$ fragmentation may also have an impact for the other production channels where delayed accuracy of the parton-fragmentation approximation is observed: $\underline{8}^1S_0$? $\underline{8}^3P_0$?
- ❖ $Q\bar{Q}$ fragmentation leads to predominantly longitudinal polarization in the helicity frame \Rightarrow may solve the polarization problem

PQCD Factorization Theorem

for inclusive **quarkonium** production
at next-to-leading order in m_c^2/p_T^2

New factorization formula

motivates complete reorganization of QCD calculations

$$\begin{aligned} d\sigma[H] = & \sum_i d\hat{\sigma}[i] \otimes D[i \rightarrow H] && \text{LO in } m_c/p_T \\ & + \sum_m d\hat{\sigma}[Q\bar{Q}_m] \otimes D[Q\bar{Q}_m \rightarrow H] && \text{order } m_c^2/p_T^2 \\ & + d\sigma_{\text{direct}}[H] && \text{order } m_c^4/p_T^4 \end{aligned}$$

To make predictions with LO (NLO) accuracy at all p_T ,
cross sections and **fragmentation functions**
should all be calculated to LO (NLO) in α_s

New factorization formula: cross sections

- single-parton cross sections
already available (LO and NLO in α_s)
- collinear $Q\bar{Q}$ cross sections
LO: Kang, Qiu & Sterman?
NLO?
- direct cross sections
already calculated to NLO,
but fragmentation terms must be consistently subtracted

New factorization formula:

fragmentation functions

- parton fragmentation functions

LO in α_s :

S-waves Braaten, Cheung, & Yuan 1993; Braaten and Yuan 1993, 1995

P-waves Braaten and Yuan 1994; Yuan 1994; Chen 1994; Ma 1995;
Hao, Zuo & Qiao 2009

D-waves Cho & Wise 1995; Cheung & Yuan 1996;
Qiao, Yuan & Chao 1997

NLO:

$g \rightarrow \underline{8}^3S_1$ Braaten & Lee 2004

$c \rightarrow \underline{1}^3S_1$ Gong, Li & Wang 2011

- $Q\bar{Q}$ fragmentation functions

LO in α_s : Kang, Qiu & Sterman

NLO?

New factorization formula:

fragmentation functions

- parton fragmentation functions

LO in α_s :

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- $Q\bar{Q}$ fragmentation functions

LO in α_s : Kang, Qiu & Sterman

NLO?

We are currently working
on some other channels

Part II. Gluon fragmentation into charmonium at NLO

Fragmentation function: formal definition

- early calculation at LO:

fragmentation functions for heavy quarkonium were extracted by comparing fixed-order cross sections with the form predicted by the factorization theorem

Fragmentation function: formal definition

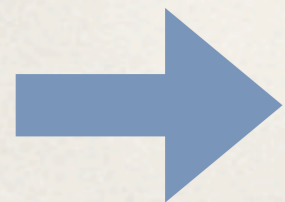
- fragmentation functions can also be defined formally as matrix elements for non-local gauge-invariant operators

Collins & Soper 1982

$$D_{g \rightarrow H}(z, \mu) = \frac{-z^{d-3}}{16\pi(d-2)k^+} \int dx^- e^{-ik^+ \cdot x^-} \\ \times \langle 0 | G_c(0)^{+\mu} \mathcal{E}^\dagger(0^-)_{cb} \mathcal{P}_{H(zk^+, 0_\perp)} \mathcal{E}(0^-)_{ba} G_a(0^+, x^-, 0_\perp)_\mu^+ | 0 \rangle$$

with the line-integral defined as

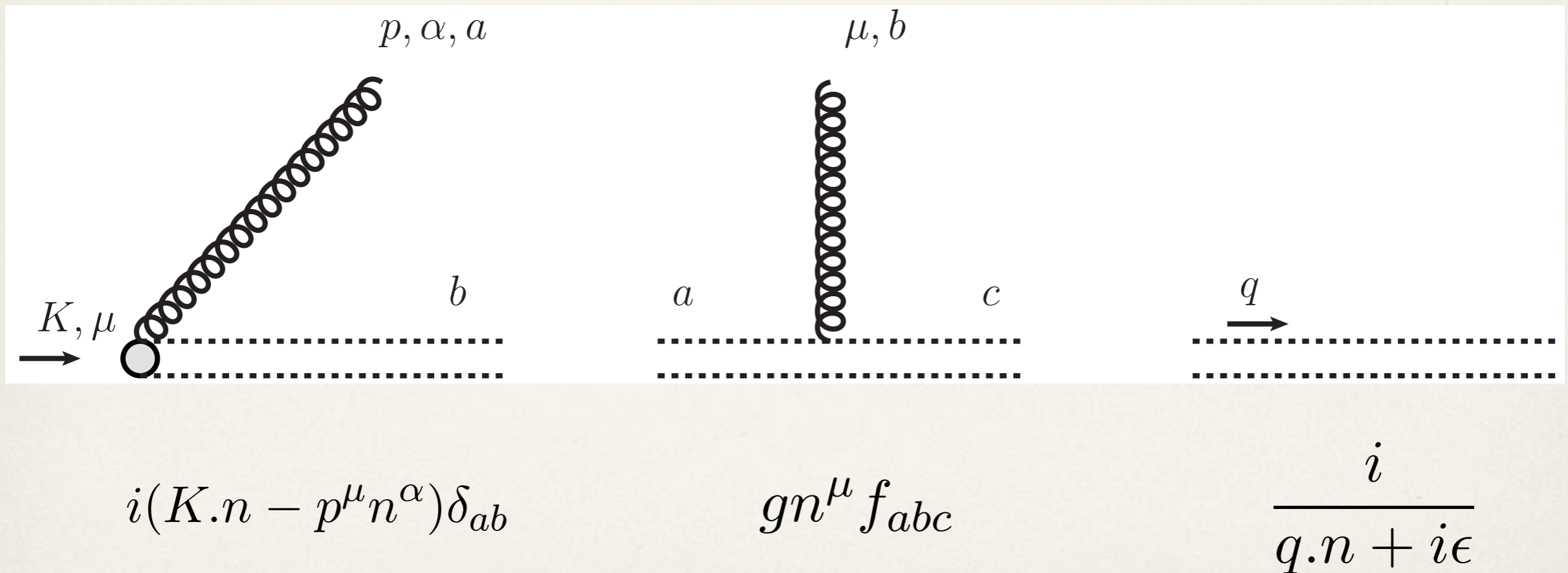
$$\mathcal{E}^\dagger(0^-)_{ba} = P \exp \left[ig \int_{x^-}^{\infty} dz^- A^+(0^+, z^-, 0_\perp) \right]_{ba}$$



the calculation of radiative corrections can be simplified by using the Feynman gauge

Fragmentation function: formal definition

- The **perturbative expansion** of this definition in powers of α_s leads to a simple set of **Feynman rules** involving the eikonal line

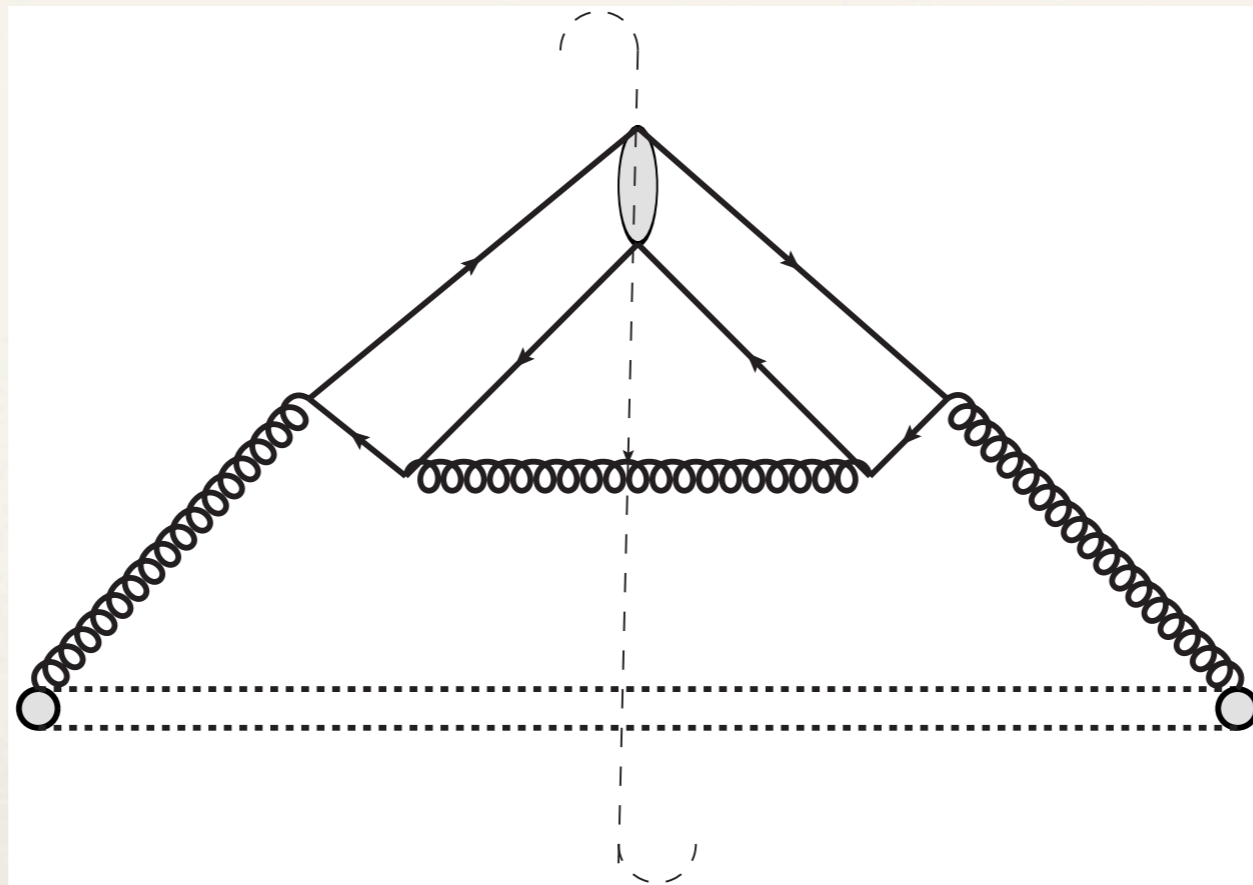


Gluon fragmentation into S-wave

- the fragmentation function $g \rightarrow \underline{8} \ ^3S_1$ is already known at NLO accuracy Braaten & Lee, 2004

- next step: gluon fragmentation into S-wave spin-singlet

Leading-order: 4 cut diagrams



+ 3 other cut diagrams

NLO correction: strategy

- Dimensional regularization ($D=4-2\epsilon$)
- Avoid the projection method, since the projector onto spin-singlet involves the Dirac matrix γ_5
- Reduce the **real** and **virtual** amplitudes to a minimal set of **scalar integrals** (FeynCalc)
- Extract the **UV/IR poles** analytically
- UV poles cancelled in the **$\overline{\text{MS}}$ scheme** (renormalization of the non-local operator, the coupling constant and the heavy quark mass)

NLO correction: strategy

- Dimensional regularization ($D=4-2\epsilon$)
- Avoid the projection method since the projector onto spin-singlet involves the γ_5
- Reduce the **real** and **scalar integrals** (Feynman diagrams) to a minimal set of
- Extract the **U** **STILL UNDER WORK**
- UV poles cancelled in the $\overline{\text{MS}}$ scheme (renormalization of the non-local operator, the coupling constant and the heavy quark mass)



Summary

PQCD factorization theorem for inclusive hadron production at **large p_T** has been extended to **quarkonium** production, including terms that are NLO in **m_Q^2/p_T^2**

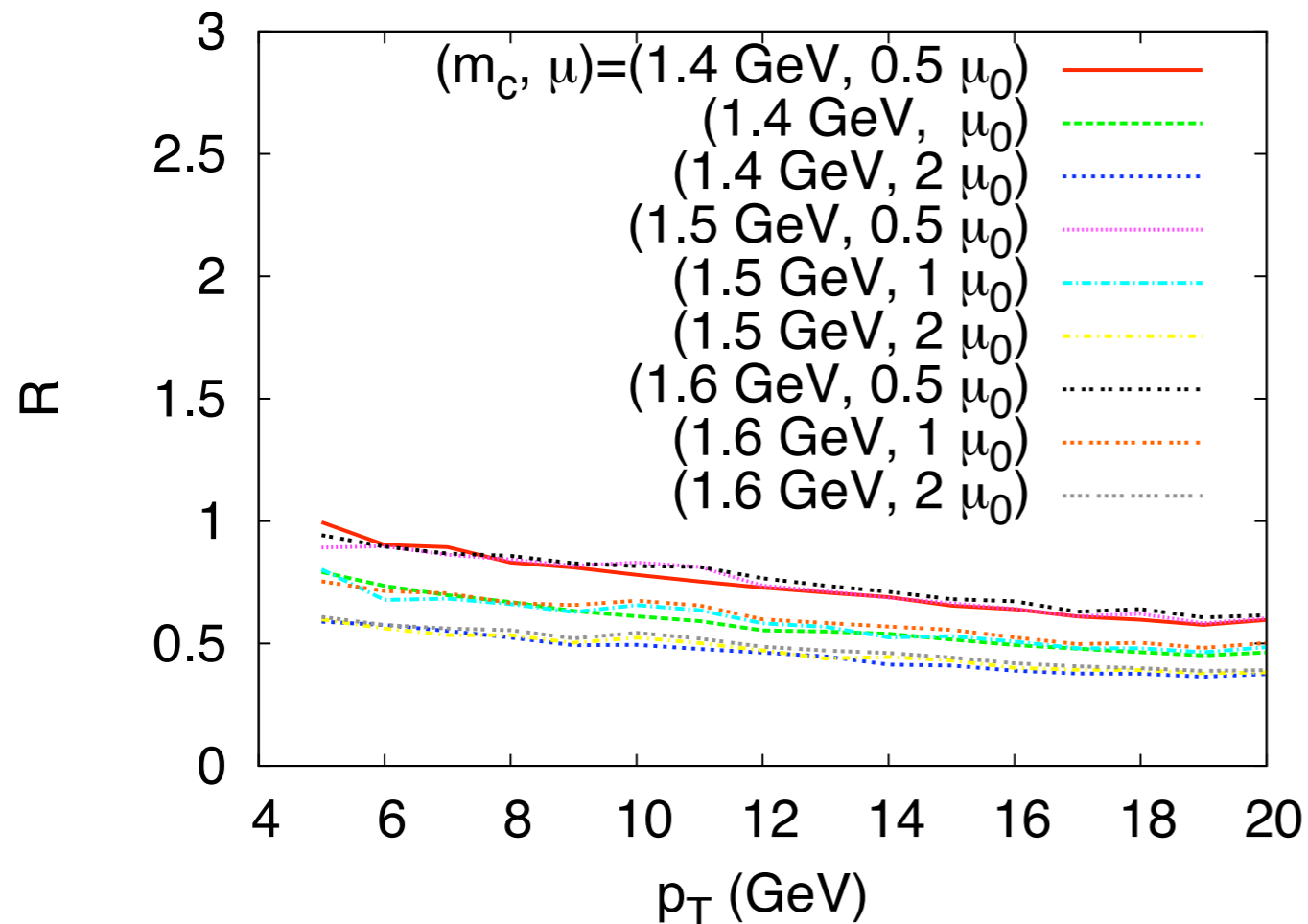
Kang, Qiu & Sterman

Factorization formula involves **nonperturbative fragmentation functions** that can probably be reduced to **constants** by using **NRQCD factorization**

Phenomenological implications on the quarkonium production at **LO** and **NLO** accuracy need to be investigated

Back-up slides

D[g → 8 ³S₁]: DGLAP evolution

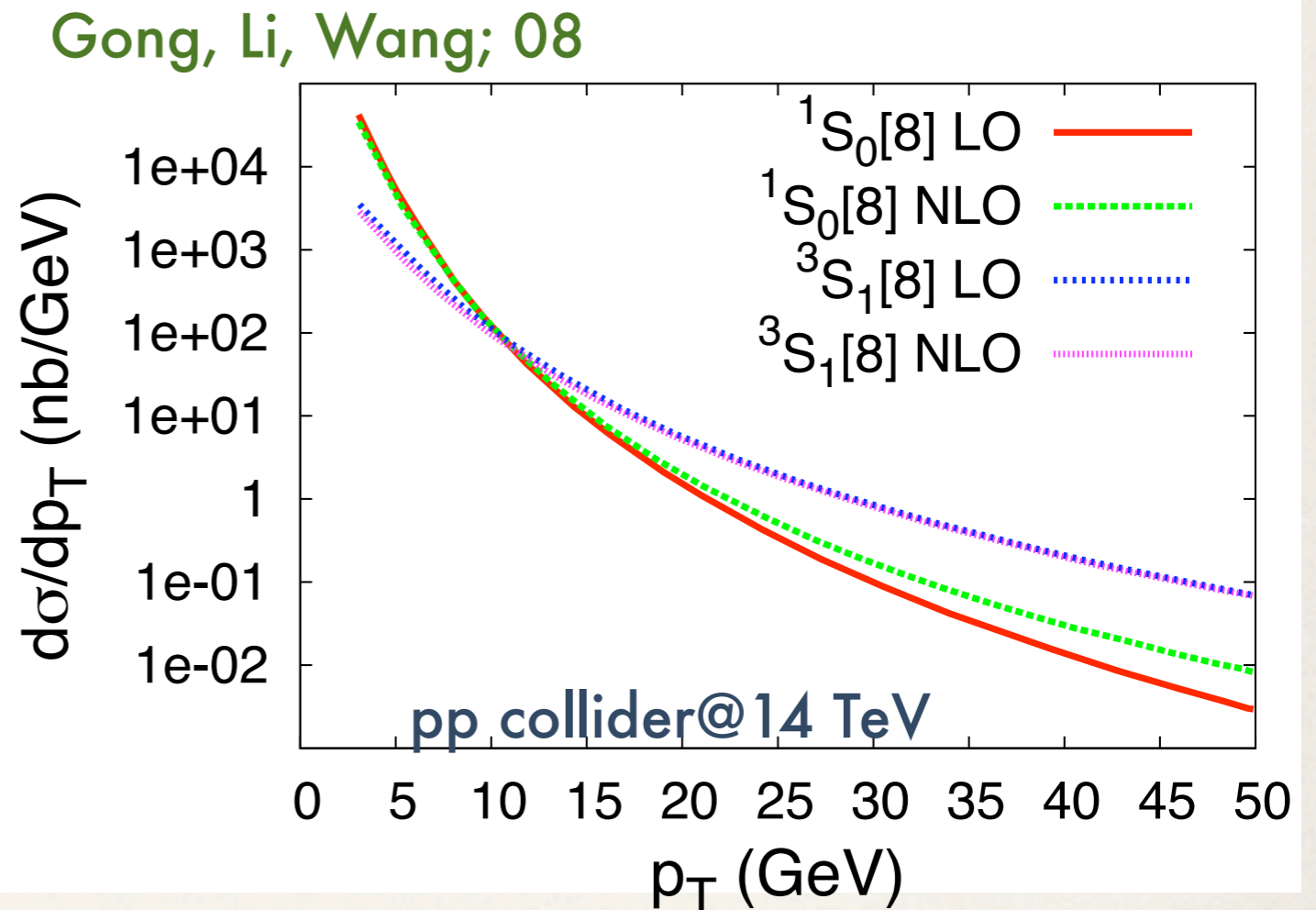


$$R = \frac{d\sigma^{\text{frac}}/dp_T(\mu_{\text{fr}} = \mu_r)}{d\sigma^{\text{frac}}/dp_T(\mu_{\text{fr}} = 2m_c)}$$

The impact of the evolution is to decrease $d\sigma/dp_T$ by a factor ≈ 2 at $p_T = 20 \text{ GeV}$

NLO correction to color-octet 3S_1

NLO correction to color-octet 3S_1 is **very small** over the entire p_T range



Question: why don't we see any effects of the large $\log(p_T/m_c)$ at high p_T ?